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Black Holes and Sub-millimeter Dimensions

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Abstract

Recently, a new framework for solving the hierarchy problem was proposed which does not rely on low energy supersymmetry or technicolor. The fundamental Planck mass is at a TeV and the observed weakness of gravity at long distances is due the existence of new sub-millimeter spatial dimensions. In this letter, we study how the properties of black holes are altered in these theories. Small black holes—with Schwarzschild radii smaller than the size of the new spatial dimensions—are quite different. They are bigger, colder, and longer-lived than a usual $(3+1)$-dimensional black hole of the same mass. Furthermore, they primarily decay into harmless bulk graviton modes rather than standard-model degrees of freedom. We discuss the interplay of our scenario with the holographic principle. Our results also have implications for the bounds on the spectrum of primordial black holes (PBHs) derived from the photo-dissociation of primordial nucleosynthesis products, distortion of the diffuse gamma-ray spectrum, overclosure of the universe, gravitational lensing, as well as the phenomenology of black hole production. For example, the bound on the spectral index of the primordial spectrum of density perturbations is relaxed from 1.25 to 1.45-1.60 depending on the epoch of the PBH formation. In these scenarios PBHs provide interesting dark matter candidates; for 6 extra dimensions MACHO candidates with mass $\sim 0.1M_\odot$ can arise. For 2 or 3 extra dimensions PBHs with mass $\sim 2000M_\odot$ can occur and may act as both dark matter and seeds for early galaxy and QSO formation.

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1 Introduction

Recently, Arkani-Hamed *et al.* [1, 2, 3] proposed a new framework for solving the hierarchy problem which does not rely on supersymmetry or technicolor. The hierarchy problem is solved by bringing the fundamental Planck scale, where gravity becomes comparable in strength to the other interactions, down to the weak scale. The observed weakness of gravity at long distances is due to the presence of n new spatial dimensions large compared to the electroweak scale. This follows from Gauss' Law which relates the Planck scales of the $(4+n)$-dimensional theory M_* and the long-distance 4-dimensional theory M_{pl},

$$M_{pl}^2 \sim R^n M_*^{n+2}, \tag{1}$$

where R is the size of the extra dimensions. Putting $M_* \sim 1$ TeV then yields

$$R \sim 10^{\frac{30}{n}-17}\text{cm}. \tag{2}$$

For $n = 1$, $R \sim 10^{13}$ cm, so this case is excluded since it would modify Newtonian gravity at solar-system distances. Already for $n = 2$, however, $R \sim 1$ mm, which is precisely the distance where present experimental measurements of gravitational strength forces stop. As n increases, R approaches $(\,\text{TeV})^{-1}$ distances, albeit slowly: the case $n = 6$ gives $R \sim (10\,\text{MeV})^{-1}$.

While the gravitational force has not been measured beneath a millimeter, the success of the SM up to $\sim 100\,\text{GeV}$ implies that the SM fields can not feel these extra large dimensions; that is, they must be stuck on a wall, or "3-brane", in the higher dimensional space. Summarizing, in this framework the universe is $(4+n)$-dimensional with Planck scale near the weak scale, with $n \geq 2$ new sub-millimeter sized dimensions where gravity and perhaps other fields can freely propagate, but where the SM particles are localised on a 3-brane in the higher-dimensional space.

An important question is the mechanism by which the SM fields are localised to the brane. The most attractive possibility is to embed in type I or type II string theory using the D-branes that naturally occur [2, 4]. This has the obvious advantage of being formulated within a consistent theory of gravity, with the additional benefit that the localization of gauge theories on a 3-brane is automatic [4]. Of course, the most pressing issue is to ensure that this framework is not experimentally excluded. This was the subject of [3] where phenomenological, astrophysical and cosmological constraints were studied and found not to exclude the framework.

There are a number of important papers with related ideas which concern themselves with the construction of string models with extra dimensions larger than the string scale

[5, 6], and with gauge coupling unification in higher dimensions [7, 8, 9]. There are also significant papers by Sundrum on the effective theory of the low energy degrees of freedom in realizations of the world-as-a-brane [10].

The objective of the present paper is to study one of the model independent aspects of the new framework, namely that gravity is altered at distances less than the size of the new dimensions. Since these distances are always less than a millimeter, this change—as explained in reference [3]—is not important for normal stars or for neutron stars. However, cosmologically interesting black holes have routinely sub-millimeter sizes and therefore their properties can be drastically altered if there are new sub-millimeter dimensions. For example, traditional 4-dimensional black holes evaporating today weigh as much as Mount Everest and are a fermi across. Their properties and signatures change radically in our framework. In section 2 we discuss the microphysical properties of such "small" black holes, including the interplay of the world-as-a-brane scenario with the holographic principle. In the rest of the paper we discuss possible observational implications for cosmology and astrophysics, which include: implications for the bounds on the spectrum of primordial black holes (PBHs) derived from the photo-dissociation of BBN products, distortion of the diffuse gamma-ray spectrum, overclosure of the universe, gravitational lensing, as well as the phenomenology of black hole production. For example, the bound on the spectral index N of the primordial spectrum of density perturbations is relaxed from $N = 1.25$ to $\sim 1.45 - 1.6$ depending on the epoch of the PBH formation. In these scenarios PBHs provide interesting dark matter candidates; further, in the cases of $n = 2$ or 3 extra dimensions PBHs with mass $\sim 2000 M_\odot$ arise naturally and may act as both dark matter and possibly even seeds for early galaxy and QSO formation, although we are not able to examine the physics of this last suggestion in any detail.

2 Properties of Small Black Holes

When the Schwarzschild radius of a black hole is much smaller than the radius R of the compactified dimensions, it should be insensitive to the brane and the boundary conditions in the n transverse dimensions, and so is well approximated by a $(4+n)$-dimensional Schwarzschild black hole. As we increase its size at some point its radius exceeds R and the black hole should go over to an effective description as a 4-dimensional black hole. Estimates made below will use the value $M_{pl} = 10^{19}$ GeV for the 4-dimensional Planck scale, and for the $(4+n)$-dimensional Planck scale, M_*, values varying between 1 TeV for $3 \leq n \leq 6$ and, for the $n = 2$ case, the astrophysically preferred value $M_* = 10\,\mathrm{TeV}$ (see [3] for discussion of the bounds on M_*).

We can understand the cross-over behavior more quantitatively by recalling some basic properties of Schwarzschild black holes in $4+n$ dimensions. We start with the size of small black holes. Following Laplace [11] we can estimate the horizon radius $r_{s(4+n)}$ of a black hole of mass M by equating the kinetic energy of a particle moving at the speed of light with its gravitational binding energy:

$$\frac{mc^2}{2} \sim \frac{GMm}{r_{s(4+n)}^{1+n}}. \tag{3}$$

Setting c and $\hbar$ to 1, and using the $(4+n)$-dimensional relation between Newton's constant and the Planck scale,

$$G = \frac{1}{M_*^{2+n}}, \tag{4}$$

gives

$$r_{s(4+n)} \sim \frac{1}{M_*}\left(\frac{M}{M_*}\right)^{\frac{1}{n+1}}. \tag{5}$$

Using the exact Schwarzschild solution in higher dimensional general relativity [12], modifies this relation only by numerical factors:

$$r_{s(4+n)} = \frac{1}{M_*}\left(\frac{M}{M_*}\right)^{\frac{1}{n+1}} \cdot \left(\frac{8\Gamma((n+3)/2)}{(n+2)\pi^{(n+1)/2}}\right)^{1/(n+1)}. \tag{6}$$

This is to be compared with the four-dimensional Schwarzschild radius

$$r_{s(4)} \sim \frac{1}{M_{pl}}\left(\frac{M}{M_{pl}}\right) \sim \frac{1}{M_*}\left(\frac{M}{M_*}\right)\frac{1}{(M_*R)^n}, \tag{7}$$

where we have used (1). Thus we have the relation

$$\left(\frac{r_{s(4)}}{r_{s(4+n)}}\right) \sim \left(\frac{r_{s(4+n)}}{R}\right)^n. \tag{8}$$

The derivation of this relation is valid when $r_{s(4+n)} \leq R$. We immediately learn that when $r_{s(4)} < R$ then

$$r_{s(4)} < r_{s(n+4)} < R. \tag{9}$$

This confirms that the cross-over behavior between 4 and $(4+n)$-dimensional black holes takes place smoothly, and implies that small enough black holes of a given mass will be larger in a brane universe than otherwise. The mass M_{cr} of a black hole right at the cross-over region where $r_{s(4)} \sim r_{s(n+4)} \sim R$ is, from (5) and (1),

$$M_{cr} \sim M_{pl}\left(\frac{M_{pl}}{M_*}\right)^{1+\frac{2}{n}} \sim 10^{\frac{30}{n}-23} M_{\odot} \tag{10}$$

which ranges from about a hundredth of an earth mass for two extra dimensions to that of a large building for large n.

Note that we have assumed that the brane tension itself does not strongly perturb the $(4+n)$-dimensional black hole solutions that we use by more than $O(1)$ factors. We can argue this as follows (specializing to the case of $n = 2$ for simplicity—the details are different for $n > 2$, but the overall conclusions are unaltered): The presence of the 3-brane in the 2-dimensional transverse space has the effect of producing a conical singularity in the transverse space at the site of the brane with deficit angle $\delta = f^4/4M_*^4$ (see Ref. [10] for details). The singularity itself is most likely resolved by the TeV-scale string theory that underlies the scenario, but most importantly at distances long compared to $(1\,\mathrm{TeV})^{-1}$ the *only* effect of the brane is the deficit angle. Thus in the "exact" $(4+2)$-dimensional formulae the area of the $S^{(n+2)}$ sphere $A(S^{(n+2)})$ should actually be replaced by $(1-\delta/2\pi)A(S^{(n+2)})$. But the deficit $\delta/2\pi$ must be small for the consistency of the world-as-a-brane scenario. The general statement for any n is that the curvature radius of the internal dimensions must be larger than their extent, namely, R. Moreover the dependence of the Schwarzschild radius of the $(4+n)$-dimensional black hole on its mass M and on M_* is unaltered by the presence of the brane. We ignore these $O(1)$ correction factors in our discussion.

The Hawking temperature $T_{(4+n)}$ of a $(4+n)$-dimensional black hole can be easily estimated from the first law of black hole thermodynamics

$$T_{(4+n)} = \frac{dE}{dS} \sim \frac{dM}{dA} \sim \frac{M}{(r_{s(4+n)}M_*)^{n+2}} \sim M_* \left(\frac{M_*}{M}\right)^{\frac{1}{n+1}}. \tag{11}$$

Using the exact Schwarzschild solution in higher dimensional general relativity [12] again modifies this relation only by numerical factors:

$$T_{(4+n)} = M_* \left(\frac{M_*}{M}\right)^{\frac{1}{n+1}} \cdot \left(\frac{(n+1)^{n+1}(n+2)}{2^{2n+5}\pi^{(n+1)/2}\Gamma((3+n)/2)}\right)^{1/(n+1)}. \tag{12}$$

From (10) the Hawking temperature for cross-over black holes goes as

$$T_{cr} \sim M_*(M_*/M_{pl})^{2/n}. \tag{13}$$

Compared to the temperature $T_{(4)} \sim M_{pl}^2/M$ of a 4-dimensional black hole of the same mass, (11) shows that small black holes with mass $M < M_{cr}$ are cooler:

$$T_{(4)} > T_{(4+n)} > T_{cr}. \tag{14}$$

This follows intuitively from the fact that they are larger, and therefore have a larger entropy (area) for the same energy (mass).

The lifetime of a small black hole is correspondingly longer than that of an equal mass 4-dimensional one. The lifetime is estimated from

$$\frac{dE}{dt} \sim (\text{Area}) \cdot T^{4+n} \tag{15}$$

giving from (5) and (11)

$$\tau_{(4+n)} \sim \frac{1}{M_*}\left(\frac{M}{M_*}\right)^{\frac{n+3}{n+1}}, \tag{16}$$

to be compared with the lifetime of a 4-dimensional black hole

$$\tau_{(4)} \sim \frac{1}{M_{pl}}\left(\frac{M}{M_{pl}}\right)^{3}. \tag{17}$$

Again, for $M < M_{cr}$ we find,

$$\tau_{(4)} < \tau_{(4+n)} < \tau_{cr}. \tag{18}$$

Again, a more precise lifetime is derived using the higher dimensional relationship between temperature and energy density,

$$\rho(T) = g_* T^{n+4}\,\frac{(n+3)\Gamma((n+4)/2)\zeta(n+4)}{\pi^{(n+4)/2}}, \tag{19}$$

where ζ is the standard Riemann zeta-function, and g_* is the number of effectively massless degrees of freedom in the bulk. In the minimal scenario in which only higher-dimensional gravity propagates in the bulk, g_* is just the number of polarization states of the $(4+n)$-dimensional graviton

$$g_* = \frac{(n+4)(n+1)}{2}. \tag{20}$$

The lifetime is then

$$\begin{aligned}\tau_{(n+4)} \;=\; & \frac{1}{M_*}\left(\frac{M}{M_*}\right)^{\frac{n+3}{n+1}} \cdot \left(\frac{2^{2n^2+9n+13}\Gamma((n+3)/2)^{n+3}}{(n+2)^2}\right)^{1/(n+1)} \\ & \cdot \left(\frac{\pi^{(2n+7)/2}}{(n+1)^{n+4}(n+3)^2\Gamma((n+6)/2)\zeta(n+4)}\right).\end{aligned} \tag{21}$$

This last calculation implicitly assumed that the small black hole was radiating in the bulk (off the brane) where it cannot emit any of the particles localized to the brane. If the black hole happened to intersect the brane, its emission rate would be enhanced because of the extra SM brane modes. However this enhancement of radiation channels on the brane as compared to the bulk gives a factor of at most 20 and must be compared with the drastically reduced phase space for radiating in the brane. Since the width of the brane is less than or on the order of M_*^{-1}, the phase space suppression factor is less than or on the order of $(M_* R)^{-n} \sim (M_*/M_{pl})^{-2} \sim 10^{-30}$ at cross-over. We should note that this conclusion depends on our assumption that the energy-density due to the brane tension does not drastically distort the bulk black hole solution.

Also note that in the standard 4-dimensional picture the smallest mass BH that one can discuss before having to worry about quantum gravity, or string theory, is $M = M_{pl} = 1.2 \times 10^{19}\,\mathrm{GeV}$. But in our picture one can describe black holes by semi-classical physics down to much smaller masses of order the fundamental Planck scale $M = M_* = 1\,\mathrm{TeV}$.

2.1 State counting and implications for holography

A fundamental principle of 4-dimensional BH physics asserts that there is one quantum state per Planck area. The natural generalization to $(4+n)$-dimensions is that there is one quantum state per $(2+n)$-dimensional Planck volume. However, the fundamental Planck length is now $O(\mathrm{TeV}^{-1})$, which is 16 orders of magnitude bigger than the usual 4-dimensional Planck length. Therefore one might think that there are far fewer degrees of freedom in our framework, and, for example the holography principle [13] becomes much more restrictive. This is not the case.

To see this let us first consider a big black hole with Schwarzschild radius larger than the size of the extra dimensions. What is the entropy of such a big BH calculated from the $(4+n)$-dimensional perspective? Taking the transverse dimensions to be an n-torus, the horizon of a large black hole would be approximately an $S^2 \times T^n$ with the S^2 of area $r_{s(4)}^2$ and the T^n of volume R^n. The entropy is proportional to the total area of the horizon in $(4+n)$-dimensional Planck units:

$$S \sim A \sim r_{s(4)}^2 R^n M_*^{n+2}. \tag{22}$$

But using Eqn. (1) this is

$$S \sim r_{s(4)}^2 M_{pl}^2, \tag{23}$$

exactly the entropy of the black hole as calculated from the 4-dimensional effective field theory perspective! The reason for this is that although the unit volume has vastly increased, we can now fill up the volume of the extra dimensions with quantum states, and the two effects precisely compensate each other as they should. Similarly the temperatures of a large BH as calculated from the two perspectives agree

$$T_{(4)} \sim M/(r_{s(4)}^2 R^n M_*^{n+2}) \sim M_{pl}^2/M. \tag{24}$$

For small BH's of radius less than R, the 4- and $(4+n)$-dimensional entropies of a BH of a given mass are related by

$$S_{(4+n)} = S_{(4)} \left(\frac{R}{r_{s(4)}}\right)^{n/(n+1)} . \tag{25}$$

Since we have shown $r_{s(4)} < r_{s(4+n)} < R$, the $(4+n)$-dimensional entropy is always larger than the 4-dimensional one. Thus at distances less than R there are more quantum states available, rather than fewer. Finally, one may wonder what happens to the 4 vs. $(4+n)$-dimensional pictures when $M \sim M_*$, and $r_{s(4+n)} \sim 1/M_*$. At this mass the $(4+n)$-dimensional BH has entropy $S_{(4+n)} = O(1)$, and roughly 1 quantum state. This does not conflict with any 4-dimensional results since for as small a mass as 1 TeV the usual 4-dimensional BH is very far below the mass where it can be described by anything other than the full theory of quantum gravity (if the BH exists at all).

In summary, for large BH's the holography constraints coincide in 4 and $(4+n)$ dimensions, as they should, whereas for small BH's the holography constraints are less restrictive in $(4+n)$ dimensions despite the fact that the fundamental Planck volume is now much bigger.

3 Phenomenology of small black holes

All black holes, no matter what their initial mass, in the end shrink down to a size where they must be described by the $(4+n)$-dimensional results already discussed. Thus in the world-as-a-brane scenario black holes are longer lived than they otherwise would be. The most important changes, however, involve the formation and decay of black holes. We now turn to an examination of how these modified properties of small black holes in a brane universe affect various black hole bounds on cosmological parameters. These bounds can be divided into three categories roughly concerned with (1) the decay of small black holes, (2) the production of primordial black holes, and (3) the present mass density of black holes. We will address these in turn.

3.1 Decay

The bounds coming from the decay of black holes are enormously weakened in a brane universe. There are two factors which affect the decay of black holes in a brane universe. First, as we saw in the last section, brane universe black holes are longer-lived than equal mass black holes in a 4-dimensional universe would be. For example, from Eqn. (21), we find that the initial mass of black holes evaporating today is

$\simeq 5 \times 10^{-27} M_\odot$ for $n = 2$ extra dimensions, growing to $\simeq 2.4 \times 10^{-19} M_\odot$ for $n = 6$. These are to be compared to the initial 4-dimensional black hole mass $\simeq 2.5 \times 10^{-19} M_\odot$ that would be evaporating today.

The second, and much more important factor by far, is the relative suppression of black hole radiation into the brane compared to radiation into the bulk. In greater detail, if the 3-brane thickness relevant for standard model excitations is denoted by t then, for black holes smaller than the size R of the extra dimensions, the in-brane to bulk phase space suppression factor is roughly $(t/r_{s(4+n)})^n$. Certainly this is a very great suppression of the visible modes until $r_{s(4+n)}$ approaches t. At the very last moments of the black hole's existence when $r_{s(4+n)}$ nears t it is possible that visible brane modes become unsuppressed. Whether this occurs depends on whether the black hole stays attached to the brane when $r_{s(4+n)} < R$, or if it can wander off into the transverse bulk dimensions. Without knowing more details of the bulk and brane theory it is not possible to calculate the probability of such wandering in detail. Nevertheless we would expect that as its horizon size approaches t random fluctuations in its radiation (with momentum $\Delta p \sim t^{-1}$) would tend to make it leave the brane in the late stages of its life. Note, of course, that it is always possible that the black hole formed in the bulk to start with.

Even in the most conservative case where the small black hole stays localized on the 3-brane, the total amount of energy deposited over the life of the black hole into SM modes is very greatly suppressed. To see this note that the largest the 3-brane thickness t can be while still allowing a consistent phenomenology for the world-as-a-brane scenario is $t = 1\,\mathrm{TeV}^{-1}$ (in principle much thinner branes are also possible, which would give an even greater suppression). The usual constraints on the density of small black holes following from Hawking evaporation involve the emission of energetic standard-model particles such as photons which can disrupt standard cosmology. For instance photons more energetic that $\sim 1\,\mathrm{MeV}$ can disassociate big bang nucleosynthesis products, ruining the successful prediction of the light element abundances. In the usual 4-dimensional case black holes have a Hawking temperature above $1\,\mathrm{MeV}$ for a mass of $M \simeq 10^{41}\,\mathrm{GeV}$ or less. Thus such black holes can emit $O(10^{41}\,\mathrm{GeV})$ of energy into dangerous energetic modes. In contrast, even if the $(4+n)$-dimensional black hole stays fixed to the brane it emits at most $O(1\,\mathrm{TeV})$ amount of energy into standard model modes, a suppression factor of 10^{-38}. This means that as far as a brane observer is concerned, black holes decay essentially invisibly (only through gravitationally coupled modes), with a possible $O(1\,\mathrm{TeV})$ flash of γ-rays at the last instant. As a result, all of the evaporation constraints on the density of PBHs are severely weakened in the world-as-a-brane scenario. In summary, small black hole decay is non-destructive, and there are no strong limits from γ-rays, or BBN light element destruction.

The next subsection discusses the most significant remaining constraints on PBH density—those that follow from overclosure.

3.2 Production and Density Bounds

The production of small black hole of a given mass in a brane universe is easier than in a 4-dimensional universe. This follows from the fact that for $r_{s(4+n)} < R$, the $(4+n)$-dimensional Schwarzschild radius is greater than their radius as calculated with 4-dimensional gravity, $r_{s(4+n)} > r_{s(4)}$, and thus a given mass of matter on the brane has to be compressed less to form a horizon. Such brane-localized matter has a pancake-like mass distribution since its transverse extension is much smaller than its in-brane extension. Nevertheless this asymmetric collapse should lead to a black hole because such a distribution is consistent with a $(4+n)$-dimensional generalization of the hoop-conjecture for black hole formation [14] once the in-brane extent is less than $r_{s(4+n)}$.

In principle there could be many astrophysical and cosmological mechanisms that might form black holes. However, given the model dependence inherent in many of these mechanisms we will limit the discussion to the relatively minimal possibility of production via primordial density perturbations $\delta \equiv \delta\rho/\rho$, possibly resulting from some period of inflation, although the origin of the density perturbations will not affect our argument.

To start, let T_i be the maximum temperature below which the universe becomes the standard radiation dominated universe. In inflationary scenarios this is the reheating temperature after the end of inflation. The mass within the horizon at this epoch is given by

$$M_{\mathrm{H}} \simeq 0.037 \frac{M_{pl}^3}{g_*^{1/2} T_i^2}. \tag{26}$$

This is the smallest that the horizon mass can be in the era in which the universe is normal. According to standard arguments a black hole forms when a density fluctuation $1/3 < \delta\rho/\rho < 1$ enters the horizon resulting in a black hole of the horizon mass [15, 16]. Our currently observable universe contains many separate horizon regions from the time when $T = T_i$. Thus an average over the many horizon regions at $T = T_i$ determines the properties of our universe. As a result if the mean $\delta \equiv \delta\rho/\rho$ at T_i was $O(1)$ the entire universe would form black holes massively overclosing the universe now. Therefore the mean δ must be much less than unity, and PBHs are only formed by rare fluctuations of δ away from its mean and into the range $1/3 < \delta < 1$. To calculate the mass fraction of the universe in black holes we must evaluate the fraction of early horizons (at T_i) for which ρ has fluctuated into the large $\delta > 1/3$ formation range.

The assumption of a Gaussian probability distribution $P(\delta)$ for the density fluctuations $\delta(M)$ (where δ is expressed as a function of the horizon mass M at the wavelength of the perturbation)

$$P(\delta(M)) = \frac{1}{\sqrt{2\pi\sigma_{rms}^2(M)}} \exp\left(-\frac{\delta^2(M)}{2\sigma_{rms}^2(M)}\right), \tag{27}$$

enables a calculation of the initial mass fraction of black holes. Here $\sigma_{rms}(M)$ is the mass variance evaluated at horizon crossing.[2]

It is possible to correlate $\sigma_{rms}^2(M)$ on the small scales of interest with the $\sigma_{rms}^2(M)$ measured at large scales by the various cosmic microwave background experiments (COBE *etc.*). This is done by making the standard assumption of a power law behavior, of the form $k^{(N-1)}$ for the power spectrum, $N = 1$ being the scale-invariant Harrison-Zeldovich spectrum. Note that a common use of the Hawking evaporation of small PBHs is to place limits on the spectral index of models in which, as small scales are approached, the size of the density perturbations increases. Such $N > 1$ spectra are known as blue spectra and are not uncommon in the attractive hybrid inflationary models.

As discussed in Refs. [15, 16], the COBE data constrains the normalization of $\sigma_{rms}(M)$, leaving just the spectral index N of the density perturbations as a free parameter. To a sufficient approximation for our purposes, COBE implies the normalization

$$\sigma_{rms}(M) \simeq 10^{-4}\left(\frac{M}{10^{56}g}\right)^{(1-N)/4}. \tag{28}$$

The mass fraction in black holes, $\beta(M) \equiv \rho_{BH}(M)/\rho_{tot}$, is just given by the probability that a fluctuation reaches the formation range $1/3 < \delta < 1$. From the Gaussian probability distribution for $\delta(M)$, and the fact that for this distribution PBH formation is exponentially closely concentrated near the lower end of the formation range $\delta = 1/3$, the initial mass fraction is well approximated by

$$\beta_i(M) = \int_{1/3}^{1} P[\delta(M)]d\delta(M) \simeq \sigma_{rms}(M)\exp\left(-\frac{1}{18\sigma_{rms}^2(M)}\right). \tag{29}$$

[2]Mild non-gaussianity—as for example studied in the context of PBH formation in Ref. [17]—does not significantly alter the limits we find on the spectral index N. Also σ must be evaluated using a suitably defined window-function; see [15, 16] for details.

Substituting Eqn. (28) then gives the predicted initial mass fraction as a function of the spectral index N.

Black holes formed by such density fluctuations are always very massive compared to T_i, and so their density scales like non-relativistic matter. Taking account of the red-shifting of the radiation bath, the black hole mass fraction today is given by

$$\beta_0(M) = \frac{T_i}{T_{eq}} \beta_i(M) \tag{30}$$

where $T_{eq} \simeq 1\,\mathrm{eV}$ is the temperature of matter-radiation equality, and we have used the fact that the BH's of interest are cosmologically stable. The statement that mass M black holes do not currently overclose the universe is simply $\beta_0(M) < 1$. The relation Eq. (26) between the horizon mass at formation and the temperature T_i, together with Eq. (30) and the expression for $\beta_i(M)$, Eq. (29), then translates into a limit on the spectral index N, given a value for T_i, the maximum temperature at which the evolution of the universe is of the normal radiation-dominated type.

With a blue spectrum of primordial density perturbations the distribution function of PBHs is steeply concentrated at the smallest possible scales, or equivalently the highest possible temperatures. The lightest black holes that can be present with any significant number density in our universe today are thus formed immediately after the epoch of inflationary reheating. In [3, 18] a relatively conservative bound on the maximum possible reheat temperature was derived by requiring that the gravitons radiated off into the bulk by a (brane-localized) SM thermal bath not provide (an effectively non-relativistic matter) bulk mass density that would "overclose" the universe. As discussed in [18], this upper bound on the temperature T_i takes on the values $\sim (3, 5, 40, 170)\,\mathrm{MeV}$ and $0.5\,\mathrm{GeV}$ as the number of extra dimensions is varied from $n = 2$ to $n = 6$. (The analysis of [18] shows that the more stringent late photon decay constraint on the reheat temperature is automatically avoided, allowing the temperatures T_i quoted above.) This translates to a horizon mass of $\sim 2 \times 10^{60}\,\mathrm{GeV} \sim 2000 M_\odot$ and $\sim 10^{56}\,\mathrm{GeV} \sim 0.1 M_\odot$ for $n = 2$ and $n = 6$, respectively. Note that these masses, even for the $n = 6$ case are outside the range excluded by the MACHO and EROS microlensing experiments—see for example Ref. [19]. Indeed, the MACHO and EROS collaborations report several events consistent with MACHO masses in the range $M \geq 0.1 M_\odot$. Thus if the spectral index is at the limit $N = 1.47$ appropriate for $n = 6$ extra dimensions (see below), it is possible that PBHs in these world-as-a-brane scenarios could be the entire halo dark matter. An interesting further possibility that is worth mentioning is that for the lower n cases where the horizon mass is comparatively large, quasars, and maybe galaxy formation more generally, may be seeded by these massive PBHs.

This is especially interesting given the mounting evidence that substantial numbers of QSOs form surprisingly early in the evolution of the universe.

In any case, from the constraint that PBHs do not overclose the universe, together with the bounds on the maximum temperature T_i, we can calculate the bounds on the amount of blue-shifting of the spectral index N. Applying the formulae of the previous paragraphs leads to bounds lying between $N \lesssim 1.59$ for 2 extra dimensions, and $N \lesssim 1.47$ for $n = 6$. These bounds on the amount of blue-shifting are considerably *weaker* than those usually arising from $(3+1)$-dimensional PBH production via primordial density fluctuations. If the MAP and PLANCK cosmic microwave background satellite experiments measure a spectral index at the degree scale greater than 1.25 then this would favor our framework. Of course if N turns out to be less than the conventional bound 1.25 then this has no implications for the world-as-a-brane scenario, other than that the number of primordial BH's produced from primordial density fluctuations is very small.

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